Metal-insulator Transition by Holographic Charge Density Waves

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Outlines

• 1. Introduction and motivation

• 2. Holographic model of charge density waves

• 3. Summary
Introduction to holography

\[
Z_{\text{bulk}}[\phi \rightarrow \delta \phi(0)] = \left\langle \exp \left( i \int d^d x \delta \phi(0) \mathcal{O} \right) \right\rangle_{\text{QFT}}
\]

(quantum) gravitational theory

\(d+1\) dimensional spacetime

quantum field theory

\(d\) dimensional spacetime

Gravity | Field theory

\[
\begin{align*}
g_{\mu\nu} &= T^{\mu\nu} \\
A_\mu &= J^\mu \\
\phi &= \text{tr} (\Phi \Phi), \text{tr} (\Psi \Psi) \\
\psi &= \text{tr} (\Phi \Psi) \\
\phi \text{ (dilaton)} &= \text{tr} F_{\mu\nu} F^{\mu\nu}.
\end{align*}
\]
Introduction to AdS/CMT

PRL 101, 031601 (2008)
Introduction to charge density waves (CDW)

1. The Peierls transition

At finite temperatures normal electrons excited across the single-particle gap screen the electron-phonon interaction. This in turn leads to the reduction of the gap and of the magnitude of the lattice distortion, and eventually to a second-order transition at the so-called Peierls temperature $T_p$.

Rev.Mod.Phys.Vol.60(1988),No.4
2. CDW ground state

The ground state of the coupled electron-phonon system is characterized by a gap in the single-particle excitation spectrum and by a collective mode formed by electron-hole pairs involving the wave vector $q = 2k_F$.

$$\rho(\mathbf{r}) = \rho_0 + \rho_1 \cos(2k_F \cdot \mathbf{r} + \varphi)$$

This condensate is called the charge density wave (CDW)!
3. Frequency-dependent response

FIG. 8. Frequency-dependent response of the collective mode (a) without pinning, and (b) with pinning and damping. The response at frequencies $\omega > 2\Delta / \hbar$ is due to single-particle excitations.
Harmonic oscillator

\[ \frac{d^2 x}{dt^2} + \frac{1}{\tau} \frac{dx}{dt} + \omega_0^2 x = \frac{eE}{m^*} e^{i\omega t} \]

\( \omega_0 \) is the pinning frequency

\[
\text{Re}\sigma(\omega) = \frac{ne^2 \tau}{m^*} \frac{\omega^2 / \tau^2}{(\omega_0^2 - \omega^2)^2 + \omega^2 / \tau^2}
\]

\[
\text{Im}\sigma(\omega) = \frac{ne^2 \tau}{m^*} \frac{(\omega_0^2 - \omega^2) \omega / \tau}{(\omega_0^2 - \omega^2)^2 + \omega^2 / \tau^2}
\]
4. CDW in CMT

TABLE I. Various broken-symmetry ground states of one-dimensional metals.

<table>
<thead>
<tr>
<th></th>
<th>Pairing</th>
<th>Spin</th>
<th>Momentum</th>
<th>Broken symmetry</th>
<th>Low-lying collective excitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single superconductor</td>
<td>el-el</td>
<td>$S=0$</td>
<td>$q=0$</td>
<td>gauge</td>
<td>none</td>
</tr>
<tr>
<td>Triplet superconductor</td>
<td>el-el</td>
<td>$S=1$</td>
<td>$q=0$</td>
<td>gauge</td>
<td>?</td>
</tr>
<tr>
<td>Charge-density wave</td>
<td>el-hole</td>
<td>$S=0$</td>
<td>$q=2k_F$</td>
<td>translational</td>
<td>phasons</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>amplitudons</td>
</tr>
<tr>
<td>Spin-density wave</td>
<td>el-hole</td>
<td>$S=1$</td>
<td>$q=2k_F$</td>
<td>translational</td>
<td>phasons</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>magnons</td>
</tr>
</tbody>
</table>

The generation of CDW results from the *spontaneous* breaking of the translational symmetry.
5. Metal-insulator transition in condensed matter physics

• 1. Bloch-Wilson insulator by band

• 2. Anderson insulator by disorder

• 3. Peierls insulator by phonon

• 4. Mott insulator by the repulsive force between electrons
Motivation

FIG. 8. Frequency-dependent response of the collective mode (a) without pinning, and (b) with pinning and damping. The response at frequencies $\omega > 2\Delta / h$ is due to single-particle excitations.
Holographic model of charge density waves

It is essential to introduce some mechanism inducing the instability of the bulk geometry which is the spontaneous breaking of the translational symmetry.

A. Donos and J. P. Gauntlett, JHEP 1108, 140 (2011)
A. Donos, JHEP 1305, 059 (2013)
Perturbative instabilities

**Action of model** [Donos and Gauntlett, arXiv:1303.4398]

\[
S = \frac{1}{2\kappa^2} \int d^4 x \sqrt{-g} \left[ R + \frac{1}{L^2} - \frac{1}{4} t(\Phi) F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} G^{\mu\nu} G_{\mu\nu} - \frac{1}{2} (\partial_\mu \Phi \partial^\mu \Phi + m^2 \Phi^2) - \frac{1}{2} u(\Phi) F^{\mu\nu} G_{\mu\nu} \right],
\]

where \( F = dA, \ G = dB, \ t(\Phi) = 1 - \frac{\beta}{2} L^2 \Phi^2, \) and \( u(\Phi) = \frac{\gamma}{\sqrt{2}} L \Phi. \)

For simplicity below we shall set \( l^2 = 6L^2 = \frac{1}{4}, \ m^2 = -\frac{2}{l^2} = -8, \)
\( \beta = -138 \) and \( \gamma = 17.1. \)
Background I

The electrically charged AdS-RN black brane solution:

\[
 ds^2 = \frac{1}{z^2} \left( -(1 - z) f(z) dt^2 + \frac{dz^2}{(1 - z)f(z)} + dx^2 + dy^2 \right) \\
 A_t = \mu(1 - z), \quad B = 0, \quad \Phi = 0 \\
 f(z) = 4(1 + z + z^2 - \frac{z^3 \mu_1^2}{16})
\]

Chemical potential \( \mu \) 

Temperature \( \frac{T}{\mu} = \frac{48 - \mu^2}{16\pi \mu} \)
The $\text{AdS}_2 \times \mathbb{R}^2$ solution:

\[
d s_4^2 = L^2 \left( ds^2 (\text{AdS}_2) + d x_1^2 + \cdots + d x_{D-2}^2 \right)
\]
\[
F = E\text{vol} (\text{AdS}_2),
\]
\[
\phi = G = 0.
\]

coupled system of perturbations:

\[
\delta \phi = e^{-i \omega t + ik x_1} \Phi(r),
\]
\[
\delta B = e^{-i \omega t + ik x_1} \left( r^2 b_t(r) \, dt + i \omega b_r(r) \frac{d r}{r^2} \right)
\]
Substituting into the equations of motion:

\[
\left( \frac{\omega^2}{r^2} + r^2 \partial_r^2 + 2r \partial_r \right) \mathbf{\nabla} - L^2 M^2 \mathbf{\nabla} = 0
\]

\[
\mathbf{\nabla}^T = (\Phi, b_r)
\]

\[
M^2 = \begin{pmatrix}
    m_s^2 + n + s^2 + p^2 & -s (m_v^2 + p^2) \\
    -s & m_v^2 + p^2
\end{pmatrix}
\]

\[p = \frac{k}{L}\]
The matrix yields the $AdS_2$ mass spectrum:

\[
m^2_{\pm} = \frac{1}{2}(\tilde{m}_s^2 + m_v^2 + s^2) + p^2 \\
\pm \frac{1}{2} \sqrt{(\tilde{m}_s^2 - m_v^2)^2 + 2(\tilde{m}_s^2 + m_v^2 + 2p^2)s^2 + s^4}
\]

\[
\tilde{m}_s^2 = m_s^2 + n
\]

It is now straightforward to choose parameters such that there are modes violating the BF bound associated with spatially modulated phases.
Perturbative instabilities of AdS-RN black brane

\[ \delta \phi = \phi(r) \cos(kx_1) \]

\[ \delta B = b_t(r) \cos(kx_1)dt \]
Background II

The electrically charged AdS-RN black brane solution:

\[ ds^2 = \frac{1}{z^2}[-(1 - z)f(z)Qdt^2 + \frac{Sdz^2}{(1 - z)f(z)} + Vdy^2 + T(dx + z^2Udz)^2], \]

\[ A = \mu(1 - z)\psi dt, \]

\[ B = (1 - z)\chi dt, \]

\[ \Phi = z\phi, \]

\[ Q[x, 0] = S[x, 0] = T[x, 0] = V[x, 0] = \psi[x, 0] = 1, \]

\[ U[x, 0] = \chi[x, 0] = \phi[x, 0] = 0. \]
Pseudo-spectral method

By expanding the solution in terms of some sort of spectral functions, plugging it into eoms, and validating eoms at some grid points, the differential equations are replaced by a set of algebraic equations.

• The resultant solution thus has an analytical expression.
• The numerical error goes like $\propto e^{-N}$ with $N$ the number of grid points.
complemented by two caveats

• The resultant algebraic equations are generically non-linear, so here comes Newton-Raphson method.

• It turns out to be extremely time consuming to apply it in the time direction, if not impossible. Instead the finite difference methods such as Runge-Kuta or Crank-Nicolson method are often adopted.
The striped solution shows up at $T_c = 0.078 \mu$ and $k_c = 0.325 \mu$. The onset of CDW can be read off explicitly from the component of the gauge field $B_t$ by holography as

$$B_t = -\rho(x) z + O(z^2),$$

$$\rho(x) = \rho_0 + \rho_1 \cos[k_c x] + \ldots + \rho_n \cos[n k_c x] + \ldots.$$
The optical conductivity of HCDW

Turn on the fluctuations of the form $e^{-i\omega t}$ on top of the background

$$g_{\mu \nu} = \bar{g}_{\mu \nu} + h_{\mu \nu}, \quad A_{\mu} = \bar{A}_{\mu} + a_{\mu}, \quad B_{\mu} = \bar{B}_{\mu} + b_{\mu}, \quad \Phi = \bar{\Phi} + \varphi.$$ 

Solve the linear perturbation equations together with de Donder gauge and Lorentz gauge condition

$$\nabla^\mu \hat{h}_{\mu \nu} = 0, \quad \nabla^\mu a_{\mu} = 0, \quad \nabla^\mu b_{\mu} = 0$$

$$\hat{h}_{\mu \nu} = h_{\mu \nu} - h\bar{g}_{\mu \nu}/2$$
Boundary conditions

\[ b_x(x, 0) = 1, \quad a_x(x, 0) = \frac{\partial_z \chi(x, 0)}{\mu(1 - \partial_z \psi(x, 0))} \]

\[ \text{others}(x, 0) = 0. \]

\[ b_x = (1 + j_x(x)z + \ldots) e^{-i\omega t} \]

Then the homogeneous part of conductivity is given by Holography as

\[ \sigma(\omega / \mu) = \frac{4j_x^{(0)}}{i\omega} \]
• Pinned collective mode
• Gapped single particle excitation
Lorentz resonance

\[ \text{Re}\sigma(\omega) = \frac{ne^2\tau}{m^*} \frac{\omega^2/\tau^2}{(\omega_0^2 - \omega^2)^2 + \omega^2/\tau^2} \]

\[ \text{Im}\sigma(\omega) = \frac{ne^2\tau}{m^*} \frac{(\omega_0^2 - \omega^2)\omega/\tau}{(\omega_0^2 - \omega^2)^2 + \omega^2/\tau^2} \]

\[ \sigma_{\text{tot}}(\omega) = \sigma_{\text{CDW}1}(\omega) + \sigma_{\text{CDW}2}(\omega) \]
The magnitude of single particle gap is estimated as

\[ 2\Delta/T_c \approx 20.51 \]

by locating the position of the second minimum in the imaginary part of the conductivity, which is obviously much larger than the mean-field BCS value

\[ 2\Delta/T_c \approx 3.52, \]

but comparable to the values for some CDW materials such as the single crystalline TbTe3 compound whose gap is given by

\[ 2\Delta/T_c \approx 15.80. \]
Summary

• The first calculation of optical conductivity of holographic CDW, where the two fundamental features of CDW are reproduced.

• The first implementation of Peierls metal-insulator transition by a gravity dual.

• The comparability of holographic gap with real CDW materials suggests a promising window for one to understand CDW by holography.
Thanks for your attention!