Quantum metrology and detection of Unruh effect

Jieci Wang, Zehua Tian, Jiliang Jing, Heng Fan arXiv: 1405.1940
Outline

Part 1  Relativistic Quantum Information

Part 2  QFI and Metrology

Part 3  Q Metrology and U-W Detector

Part 4  Conclusions
Why RQI?

Q 2 • Why RQI?

QI

GR

不存在绝对的惯性系
比特承诺，星地量子通信

弯曲时空中的量子效应
早期宇宙，量子引力
Studies on RQI

2.1 RQI

RQI

- QI & Special Relativity
  - PRL 97, 250502 (2006)
  - PRL 98, 080406 (2007)
  - PRA 85, 042322 (2012)
  - PRL 109, 130501 (2012)
  - PRL 112, 010504 (2014)

- QI & noninertial frames
  - PRL 95, 120404 (2005)
  - PRL 106, 210502 (2011)
  - PRL 109, 033602 (2012)
  - PRL 110, 160501 (2013)
  - PRL 110, 113602 (2013)

- QI & Blackhole or Cosmology
  - PRD 78, 065015 (2008)
  - JHEP 07, 072 (2012)
  - Nat. Com. 2, 505 (2011)
  - PRD 85, 061701 (2012)
  - Science 341, 1213 (2013)
The initial state:

\[ |\Phi\rangle_{AR} = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_R + |1\rangle_A |1\rangle_R) \]

\[ |0_k\rangle_M \sim \frac{1}{\cosh r} \sum_{n=0}^{\infty} \tanh^n r |n_k\rangle_I |n_k\rangle_{II} \]

\[ \cosh r = (1 - e^{-2\pi \Omega})^{-1/2}, \quad \Omega = |k| c / a. \]

\[ |1_k\rangle_M = \frac{1}{\cosh^2 r} \sum_{n=0}^{\infty} \tanh^n r \sqrt{n+1} |(n+1)k\rangle_I |n_k\rangle_{II}. \]

FIG. 1 (color online). The negativity as a function of the acceleration \( r \).
Entanglement of Dirac fields in noninertial frames


P. M. Alsing, I. Fuentes-Schuller, R. B. Mann, and T. E. Tessier

Quantum entanglement of electromagnetic field in non-inertial reference frames


Hawking radiation, Entanglement and Teleportation in background of an asymptotically flat static black hole

Qiyuan Pan, Jiliang Jing

(Submitted on 4 Sep 2008 (v1), last revised 6 Sep 2008 (this version, v2))
Using Berry’s Phase to Detect the Unruh Effect at Lower Accelerations

Abstract

We show that a detector acquires a Berry phase due to its motion in spacetime. The phase is different in the inertial and accelerated case as a direct consequence of the Unruh effect. We exploit this fact to design a novel method to measure the Unruh effect. Surprisingly, the effect is detectable for accelerations $10^9$ times smaller than previous proposals sustained only for times of nanoseconds.

$$H_T = \Omega_a a^\dagger a + \Omega_b b^\dagger b + \lambda (b + b^\dagger)$$
$$\times [a^\dagger e^{i(kx - \Omega_a t)} + a e^{-i(kx - \Omega_a t)}], \quad (1)$$

FIG. 1. Trajectories for an inertial and accelerated detector.

FIG. 2. $\delta$ for each cycle as a function of the acceleration for three different scenarios. First scenario (top): $\Omega_a \approx 2.0$ GHz, $\Omega_b \approx 2.0$ GHz, and $\lambda \approx 34$ Hz. Second scenario (middle): $\Omega_a \approx 2.0$ GHz, $\Omega_b \approx 2.0$ GHz, and $\lambda \approx 0.10$ KHz. Third scenario (bottom): $\Omega_a \approx 2.0$ GHz, $\Omega_b \approx 2.0$ GHz, and $\lambda \approx 0.25$ KHz.
Entangling Moving Cavities in Noninertial Frames

An open question in the field of relativistic quantum information is how parties in arbitrary motion may distribute and store quantum entanglement. We propose a scheme for storing quantum information in the field modes of cavities moving in flat space-time and analyze it in a quantum field theoretical framework. In contrast with previous work that found entanglement degradation between observers moving with uniform acceleration, we find the quantum information in such systems is protected. We further discuss a method for establishing the entanglement in the first place and show that in principle it is always possible to produce maximally entangled states between the cavities.

FIG. 2 (color online). Entangling two cavities.

FIG. 4. Entanglement as a function of $L$ for $a = 8 \times 10^{38}$ m$^{-2}$. 
We study the effects of relativistic motion on quantum teleportation and propose a realizable experiment where our results can be tested. We compute bounds on the optimal fidelity of teleportation when one of the observers undergoes nonuniform motion for a finite time. The upper bound to the optimal fidelity is degraded due to the observer’s motion. However, we discuss how this degradation can be corrected. These effects are observable for experimental parameters that are within reach of cutting-edge superconducting technology.
Accelerated atoms (or detectors)

Processing Quantum Information with Relativistic Motion of Atoms

Phys. Rev. Lett. 110, 160501 – Published 15 April 2013

Eduardo Martín-Martínez, David Aasen, and Achim Kempf

We show that particle detectors, such as two-level atoms, in noninertial motion (or in gravitational fields) could be used to build quantum gates for the processing of quantum information. Concretely, we show that through suitably chosen noninertial trajectories of the detectors the interaction Hamiltonian’s time dependence can be modulated to yield arbitrary rotations in the Bloch sphere due to relativistic quantum effects.

FIG. 1 (color online). For \( \lambda|\alpha| = 0.01 \), (a) Azimuthal angle of the rotation axis for various \( T \). (b) Magnitude of the rotation \( \delta \) for these cases. Independent rotations that are more than 50° apart can be achieved by controlling the atom’s acceleration. \( \Omega \) fixes the time scale and the units of \( T \). Acceleration is expressed in natural units (we took \( c = 1 \)), so it has units of inverse time. (c) Scheme of an array of cavities with predefined coherent states \( |\alpha^1\rangle_{\omega_1} \otimes |\omega_{\neq \omega_1}\rangle_{\omega} \) that allow successive different rotations in the Bloch sphere; accelerations could alternate sign.
Unconditionally secure bit commitment is impossible if Alice and Bob are allowed to use quantum resources in a nonrelativistic scenario.
Part 2 Quantum Fisher information and metrology
Quantum metrology is the study of making high-resolution and highly sensitive estimations for physical parameters by using quantum theory, particularly exploiting quantum entanglement.

The general quantum parameter estimation procedure, e.g. phase estimation, can be divided to three parts: states preparation, evolution, and measurement.

Advances in quantum metrology

Vittorio Giovannetti\textsuperscript{1}\textsuperscript{*}, Seth Lloyd\textsuperscript{2} and Lorenzo Maccone\textsuperscript{3}

The statistical error in any estimation can be reduced by repeating the measurement and averaging the results. The central limit theorem implies that the reduction is proportional to the square root of the number of repetitions. Quantum metrology is the use of quantum techniques such as entanglement to yield higher statistical precision than purely classical approaches. In this Review, we analyse some of the most promising recent developments of this research field and point out some of the new experiments. We then look at one of the major new trends of the field: analyses of the effects of noise and experimental imperfections.
Optimal process

- The initial state is $\rho_0$, the phase is encoded on state as

$$\rho(\phi) = \mathcal{E}_\phi \circ \rho_0$$

We choose a set of POVMs $\{\Pi_\chi\}$, and the probability for outcome $\chi$ is

$$P(\chi|\phi) = \text{Tr}(\Pi_\chi \rho(\phi))$$

- The Fisher information is defined as

$$\mathcal{F}(\phi) = \sum_\chi P(\chi|\phi) \left[ \partial_\phi \ln P(\chi|\phi) \right]^2$$

- And the Cramer-Rao bound (CRB) is

$$\Delta\phi \geq 1/\sqrt{\mathcal{F}(\phi)}$$
(Quantum) Fisher Information

Classical

Probability distribution

Fisher Information

\[ F(\mu) = \int f(\partial_\mu \ln f) \, dx \]
\[ \partial_\mu f = \frac{1}{2} [f(\partial_\mu \ln f) + (\partial_\mu \ln f)f] \]

Quantum

Quantum State

\[ \rho(\mu) \]

Quantum Fisher Information (QFI)

\[ F_Q(\mu) = \text{tr}(\rho L_\mu^2) \]
\[ \partial_\mu \rho = \frac{1}{2} (L_\mu \rho + \rho L_\mu) \]

Symmetry Logarithm Derivative (SLD), Hermitian, Zero expectation value

2014-7-8
Exact form of Quantum Fisher information

$\mathcal{F}_\lambda = \text{Tr} \left( \rho_\lambda L_\lambda^2 \right) = \text{Tr} \left[ (\partial_\lambda \rho_\lambda) L_\lambda \right]$

$\partial_\lambda \rho_\lambda = \frac{1}{2} \{ \rho_\lambda, L_\lambda \}$

Mix:

$\mathcal{F}_\lambda = \sum_{i'} \frac{(\partial_\lambda Q_{i'})^2}{Q_{i'}} + 2 \sum_{i \neq j} \frac{(Q_i - Q_j)^2}{Q_i + Q_j} |\langle \psi_i | \partial_\lambda \psi_j \rangle|^2$

Pure:
Part 3  Quantum metrology and detection of Unruh effect

arXiv:1405.1940
Under the two-mode squeezing operator

$$\hat{U}(r) = \exp[r(\hat{b}_1^\dagger \hat{b}_2^\dagger - \hat{b}_1^\dagger \hat{b}_2)]$$

the vacuum state evolves to

$$\hat{\varrho}_r = N_r \sum_{k=0}^{\infty} C_r^k (\hat{b}_1^\dagger)^k (\cosh r) - \hat{b}_1^\dagger \hat{b}_2 \hat{\varrho}_0 (\cosh r) - \hat{b}_1 \hat{b}_2^\dagger \hat{b}_1^k,$$

\[\sigma_0 = \begin{pmatrix} e^{2s}\cos^2\theta + e^{-2s}\sin^2\theta & \sin 2\theta \sinh 2s \\ \sin 2\theta \sinh 2s & e^{2s}\cos^2\theta + e^{-2s}\sin^2\theta \end{pmatrix}.\]

FIG. 1 (color online). Fisher information $I_r$ corresponding, from bottom to top, to coherent states in Alice’s frame and heterodyne detection by Rob (blue), general Gaussian states in Alice’s frame and optimal detection by Rob (green), and the ultimate quantum bound, attained by Fock states in Alice’s frame and photon counting by Rob (wire frame).
Relativistic Quantum Metrology: Exploiting relativity to improve quantum measurement technologies

Mehdi Ahmadi, David Edward Bruschi, Carlos Sabín, Gerardo Adesso, Ivette Fuentes

(Submitted on 26 Jul 2013 (v1), last revised 29 Apr 2014 (this version, v3))

The transformed covariance matrix is given by,

\[
\tilde{\sigma}_{kk'} = \begin{pmatrix}
C_{kk} & C_{kk'} \\
C_{k'k} & C_{k'k'}
\end{pmatrix},
\]

where

\[
C_{ij} = M_{ij}^T \psi_k M_{kj} + M_{k'i}^T \phi_{kk} M_{kj} + M_{k'i}^T \phi_{kk'} M_{k'j} \\
+ M_{k'i}^T \psi_{k'} M_{k'j} + \sum_{n\neq i,j} M_{ni}^T M_{nj}.
\]

where the \( M_{mn} \) are the \( 2 \times 2 \) matrices

\[
M_{mn} = \begin{pmatrix}
\text{Re}(\alpha_{mn} - \beta_{mn}) & \text{Im}(\alpha_{mn} + \beta_{mn}) \\
-\text{Im}(\alpha_{mn} - \beta_{mn}) & \text{Re}(\alpha_{mn} + \beta_{mn})
\end{pmatrix}
\]

FIG. 1: (a) General cavity framework: the initial state of two modes of a quantum field inside a cavity, represented initially by the covariance matrix \( \sigma_{kk'} \) undergoes a relativistic transformation which depends on some parameter \( \Theta \). The transformed covariance matrix \( \tilde{\sigma}_{kk'} \) depends on the parameter \( \Theta \), which can be estimated using quantum metrology tools. (b) Example: measurement of the acceleration in a BEC setup.
The probe state of the estimation has the form of

$$|\Psi_{AR}\rangle = \sin \theta |0_A\rangle|1_R\rangle + \cos \theta |1_A\rangle|0_R\rangle,$$

(1)

The total initial state of the detectors plus the external scalar fields is $$|\Psi_{t_0}^{AR\phi}\rangle = |\Psi_{AR}\rangle \otimes |0_M\rangle,$$

Rob's detector moves with constant acceleration $a$ for a finite amount of time $\Delta$

$$t(\tau) = a^{-1} \sinh a\tau, \quad x(\tau) = a^{-1} \cosh a\tau,$$

$$y(\tau) = z(\tau) = 0,$$

The total Hamiltonian of the system is given by

$$H_{AR\phi} = H_A + H_R + H_{KG} + H_{int}$$
The interaction Hamiltonian between Rob’s detector and the scalar field is

$$H_{\text{int}}^{R \phi}(t) = \epsilon(t) \int_{\Sigma_t} d^3x \sqrt{-g} \phi(x) [\psi(x) R + \overline{\psi}(x) R^\dagger],$$

The state $|\Psi_{t=t_0+\Delta}^{R \phi}\rangle$ that describes Rob’s detector and the scalar field at time $t = t_0 + \Delta$ can be expressed as

$$|\Psi_t^{R \phi}\rangle = T \exp[-i \int_{-\infty}^{t} dt' H_{\text{int}}^I(t')]|\Psi_{t_0}^{R \phi}\rangle, \quad (4)$$
in the interaction picture, where $T$ is the time-ordering operator and

$$H_{\text{int}}^I(t) = U_0^\dagger(t) H_{\text{int}}(t) U_0(t). \quad (5)$$

Here $U_0(t)$ is an unitary evolution operator associated with $H_R + H_{KG}$ [27, 32]. By using Eq. (4), we write the final state $|\Psi_t^{R \phi}\rangle$ of the detector-field system as

$$|\Psi_t^{R \phi}\rangle = T \exp[-i \int d^4x \sqrt{-g} \phi(x) (f R + \overline{f} R^\dagger)]|\Psi_{t_0}^{R \phi}\rangle, \quad (6)$$

where $f \equiv \epsilon(t)e^{-i\Omega t} \psi(x)$ is a compact support complex function defined in the Minkowski space-
Quantum metrology and detection of Unruh effect

By choosing $\psi(x) = (\kappa \sqrt{2\pi})^{-3} \exp(-x^2/2\kappa^2)$ with the parameter $\kappa = \text{const} \ll \Omega^{-1}$ Eq. (6)
evolves to

$$|\Psi_{t}^{R\phi}\rangle = [I - i(\phi(f) R + \phi(f)^\dagger R^\dagger)]|\Psi_{t_0}^{R\phi}\rangle,$$

where $\phi(f)$ is an operator valued distribution of the scalar field [32] given by

$$\phi(f) \equiv \int d^4x \sqrt{-g}\phi(x)f$$
$$= i[a_{RI}(KE\bar{f}) - a_{RI}^\dagger(KEf)],$$

Considering that only Rob's detector interacts with the field, we evolve our initial state to its asymptotic form

$$|\Psi_{t}^{AR\phi}\rangle = |\Psi_{t_0}^{AR\phi}\rangle + \sin \theta |0_A\rangle |0_R\rangle \otimes (a_{RI}^\dagger(\lambda)|0_M\rangle)$$
$$+ \cos \theta |1_A\rangle |1_R\rangle \otimes (a_{RI}(\bar{\lambda})|0_M\rangle),$$

where $\lambda = -KEf$, the subscripts in $a_{RI}^\dagger$ and $a_{RI}$ indicate that they are creation and annihilation operators.
We write the operators \( a_{RI} \) and \( a_{RI}^\dagger \) as

\[
a_{RI}(\lambda) = \frac{a_M(F_1\Omega) + e^{-\pi\Omega/a}a_M^\dagger(F_2\Omega)}{(1 - e^{-2\pi\Omega/a})^{1/2}},
\]

\[
a_{RI}^\dagger(\lambda) = \frac{a_M^\dagger(F_1\Omega) + e^{-\pi\Omega/a}a_M(F_2\Omega)}{(1 - e^{-2\pi\Omega/a})^{1/2}},
\]

where \( F_1\Omega = \frac{\lambda + e^{-\pi\Omega/a}\lambda w}{(1 - e^{-2\pi\Omega/a})^{1/2}} \), and \( F_2\Omega = \frac{\lambda w + e^{-\pi\Omega/a}\lambda}{(1 - e^{-2\pi\Omega/a})^{1/2}} \). Here \( w(t, x) = (-t, -x) \) is the wedge reflection isometry, which makes a reflection from \( \varphi \in H_I \) to \( \varphi \circ w \in H_{II} \).

Now, by using \( a_{RI}(\lambda)|0_M\rangle = \frac{\nu e^{-\pi\Omega/a}}{(1 - e^{-2\pi\Omega/a})^{1/2}}|1\bar{F}_{2\Omega}\rangle \), \( a_{RI}^\dagger(\lambda)|0_M\rangle = \frac{\nu}{(1 - e^{-2\pi\Omega/a})^{1/2}}|1\bar{F}_{1\Omega}\rangle \),

we cast Eq. (9) in the form

\[
|\Psi_{\infty}^{AR\phi}\rangle = |\Psi_{-\infty}^{AR\phi}\rangle + \alpha \nu \frac{|0_A\rangle \otimes |0_R\rangle \otimes |1\bar{F}_{1\Omega}\rangle}{(1 - e^{-2\pi\Omega/a})^{1/2}}
\]

\[
+ \beta \nu e^{-\pi\Omega/a} \frac{|1_A\rangle \otimes |1_R\rangle \otimes |1\bar{F}_{2\Omega}\rangle}{(1 - e^{-2\pi\Omega/a})^{1/2}},
\]
Quantum metrology and detection of Unruh effect

The density matrix that describes the detector’s state is obtained by tracing out the degrees of freedom of the external field

$$\rho_t^{AR} = \alpha |\Psi_{AR}\rangle\langle\Psi_{AR}| + \beta |0_A\rangle\langle 0_A| + \gamma |1_A\rangle\langle 1_A|,$$

where $\Psi_{AR}$ is the initial state of the detectors and the parameters $\alpha$, $\beta$ and $\gamma$ are found to be

$$\alpha = \frac{1 - e^{-\Omega/T}}{(1 - e^{-\Omega/T}) + \sin^2 \theta \nu^2 + \nu^2 \cos^2 \theta e^{-\Omega/T}},$$

$$\beta = \frac{\nu^2 \sin^2 \theta}{(1 - e^{-\Omega/T}) + \nu^2 \sin^2 \theta + \nu^2 \cos^2 \theta e^{-\Omega/T}},$$

$$\gamma = \frac{\nu^2 \cos^2 \theta e^{-\Omega/T}}{(1 - e^{-\Omega/T}) + \nu^2 \sin^2 \theta + \nu^2 \cos^2 \theta e^{-\Omega/T}},$$

respectively, and $T = a/2\pi$ is the Unruh temperature.
Quantum metrology and detection of Unruh effect

The QFI for the estimation of $T$ can be defined by

$$\mathcal{F}_Q(T) = \text{Tr} \left( \rho_t^{AR} \mathcal{L}_T^2 \right) = \text{Tr} \left[ \partial_T (\rho_t^{AR}) \mathcal{L}_T \right],$$

(15)

Basing on a spectrum decomposition of the state as $\rho_t^{AR} = \sum_{i=1}^{N} p_i |\psi_i\rangle \langle \psi_i|$, the QFI can be rephrased as [34, 35]

$$\mathcal{F}_Q(T) = 2 \sum_{m,n}^{N} \frac{|\langle \psi_m | \partial_T \rho_t^{AR} | \psi_n \rangle|^2}{p_m + p_n},$$

(16)

with the eigenvalues $p_i \geq 0$ and $\sum_{i}^{N} p_i = 1$. For a non-full-rank state the QFI can be expressed as [36, 37]

$$\mathcal{F}_Q(T) = \sum_{m'} \frac{(\partial_T p_{m'})^2}{p_{m'}} + 2 \sum_{m \neq n} \frac{(p_m - p_n)^2}{p_m + p_n} |\langle \psi_m | \partial_T \psi_n \rangle|^2,$$

(17)
FIG. 1: (Color online) The QFI in the estimation of the Unruh temperature as functions of the coupling parameter $\nu$ and the acceleration $a$. The initial state parameter is fixed with $\theta = \pi/4$ and the energy gap is given by $\Omega = 1$. 
FIG. 2: (Color online) QFI and entanglement of the final state Eq. (12) as a function of the acceleration $a$ for a fixed effective coupling parameter $\nu = 0.8$. The initial state parameter is fixed with $\theta = \pi/4$ and the energy gap is given by $\Omega^{-1} = 2\pi$. 
FIG. 3: (Color online) QFI in the estimation of the Unruh temperature $T$ as a function of the energy gap $\Omega$ for different interaction time $\Delta$. The parameters related to the effective coupling parameter are fixed to satisfy $\epsilon \ll \Omega^{-1} \ll \Delta$. They are fixed with $\epsilon = 2\pi \cdot 10^{-3}$ and $\kappa = 0.02$, respectively. The initial state parameter is given by $\theta = \pi/4$ and the acceleration parameter is fixed with $a = 0.4\pi$. 

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The probe state preparation and the interaction affect the value of quantum Fisher information.

Larger effective coupling strength and longer interaction time are required.

The energy gap of the detector has a range that can provide us a better precision.

An extremely high acceleration is not required.

Jieci Wang, Zehua Tian, Jiliang Jing, Heng Fan arXiv: 1405.1940
Thanks!